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[54] **COMPUTING ECONOMIC POWER DISTRIBUTION IN POWER TOOLS**

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[73] Assignee: **Leeds & Northrup Company**, Philadelphia, Pa.

[22] Filed: **Sept. 8, 1970**

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[52] **U.S. Cl.** **235/151.21**, 444/1, 307/57
[51] **Int. Cl.** **G06f 15/56**, G06f 15/06, H02j 3/06
[58] **Field of Search** 235/151.21, 150; 307/57; 444/1

[56] **References Cited**

UNITED STATES PATENTS

3,400,258 9/1968 Stadlin..................235/151.21

Primary Examiner—Eugene G. Botz
Assistant Examiner—Edward J. Wise
Attorney—William G. Miller, Jr. and Raymond F. MacKay

[57] **ABSTRACT**

The economic dispatch of generation in interconnected areas is computed by determining the generation which will cause the incremental cost of power at each of the interarea tie points, as calculated from the interconnected areas, to be equal at the existing load level. A computer having its own loss matrix is utilized in each area. It computes the tie point costs on the basis of cost information sent from the areas interconnected to it, and it also sends its own costs to those areas. In each area the desired generation and net tie line interchange are computed to provide a basis for controlling the area's generation.

6 Claims, 5 Drawing Figures

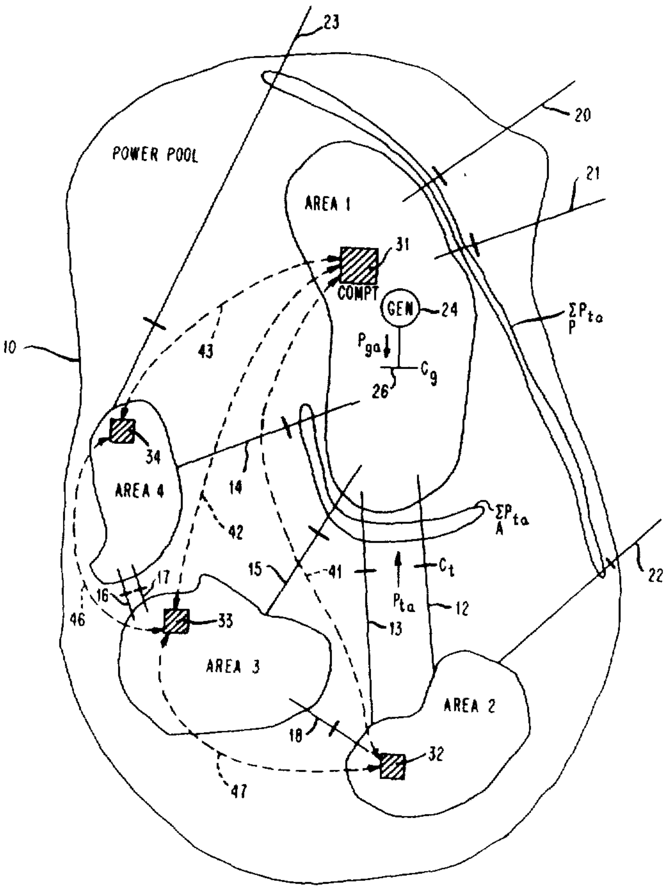

FIG. 1

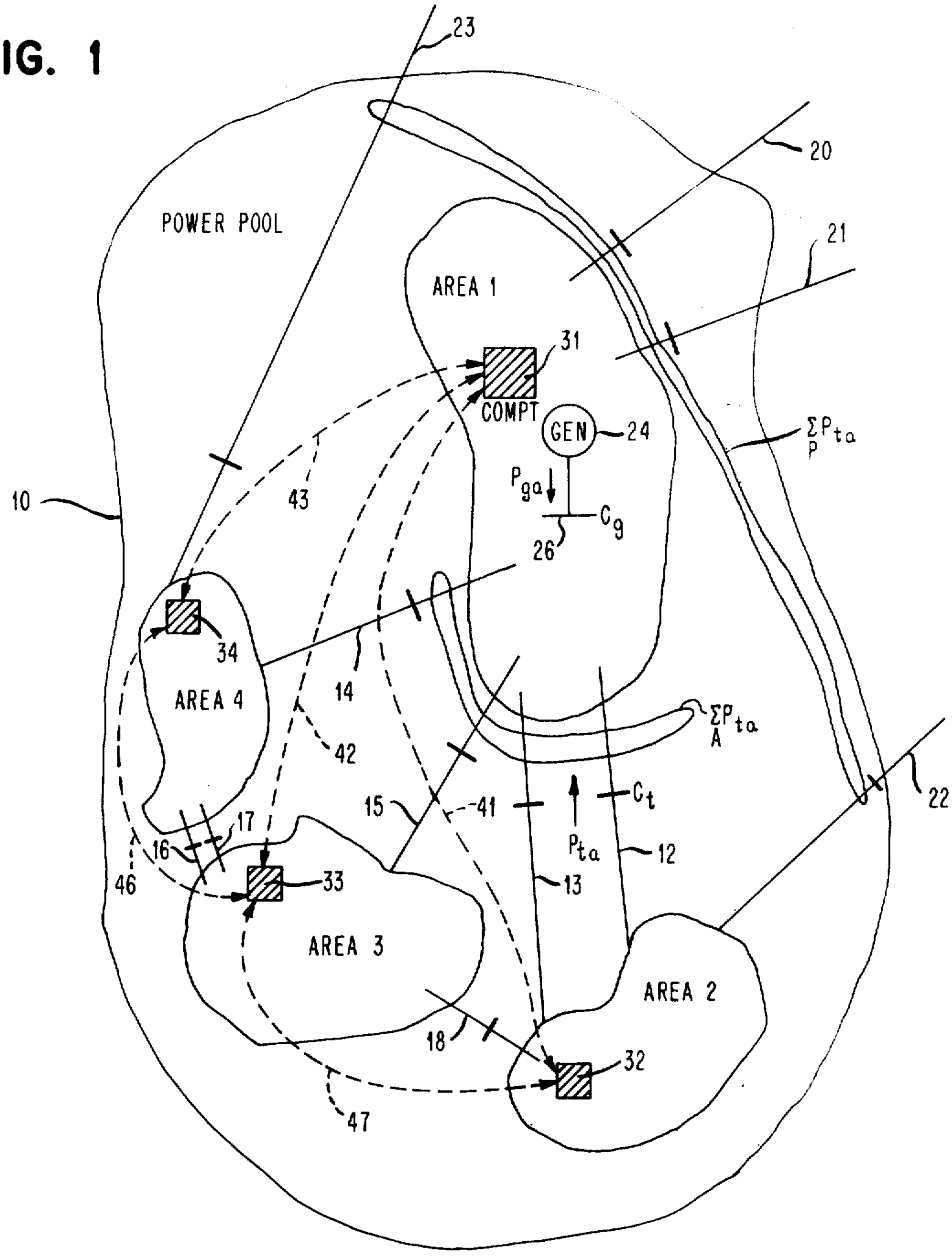

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FIG. 2

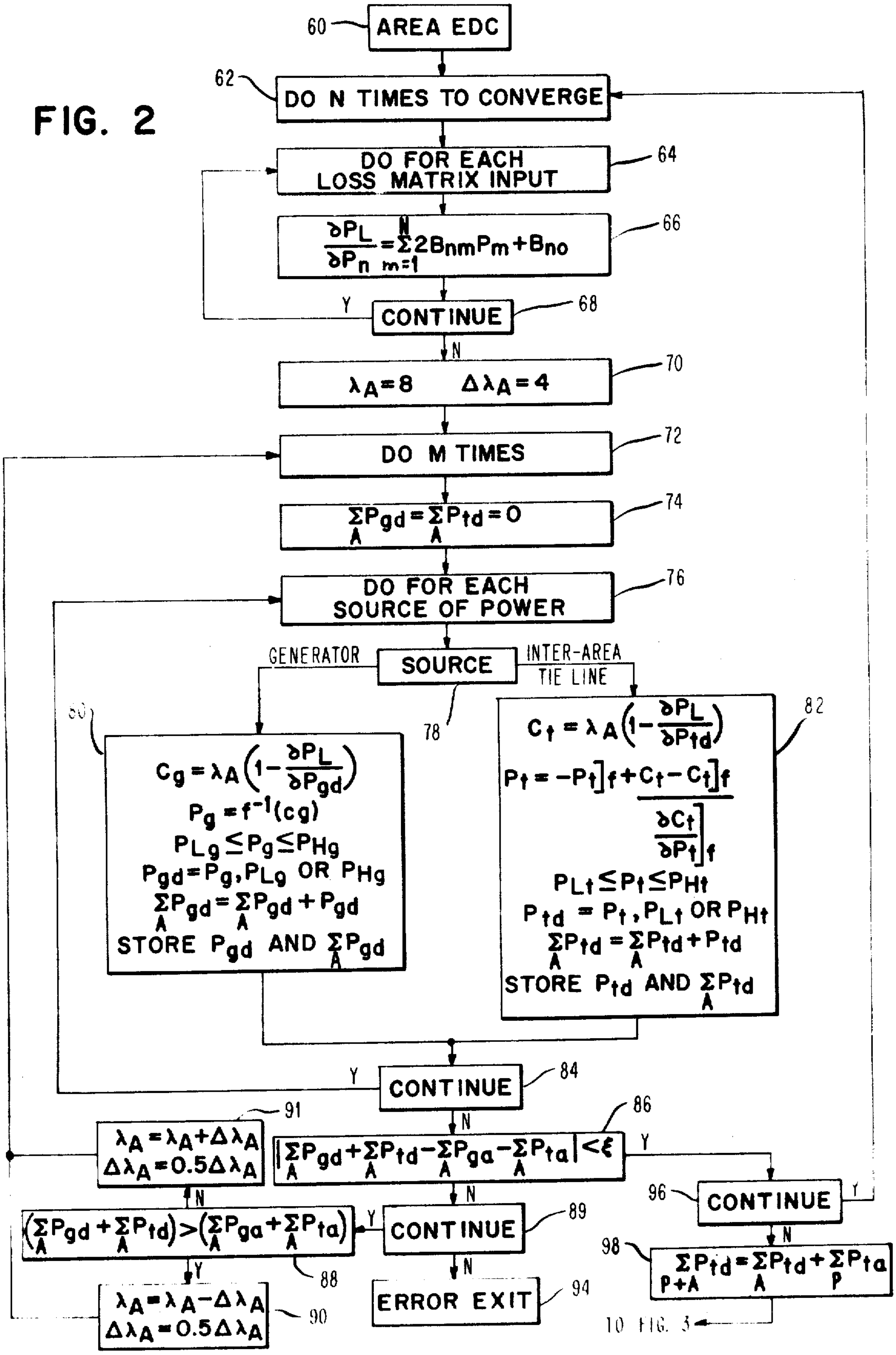

60
AREA EDC
62
DO N TIMES TO CONVERGE
64
DO FOR EACH LOSS MATRIX INPUT
66
$\frac{\partial P_L}{\partial P_n} = \sum_{m=1}^{N} 2B_{nm}P_m + B_{no}$
Y
68
CONTINUE
N
70
$\lambda_A = 8 \quad \Delta\lambda_A = 4$
72
DO M TIMES
74
$\sum_A P_{gd} = \sum_A P_{td} = 0$
76
DO FOR EACH SOURCE OF POWER
GENERATOR
SOURCE
INTER-AREA TIE LINE
78
80
$C_g = \lambda_A\left(1 - \frac{\partial P_L}{\partial P_{gd}}\right)$
$P_g = f^{-1}(c_g)$
$P_{Lg} \leq P_g \leq P_{Hg}$
$P_{gd} = P_g, P_{Lg}$ OR P_{Hg}
$\sum_A P_{gd} = \sum_A P_{gd} + P_{gd}$
STORE P_{gd} AND $\sum_A P_{gd}$
82
$C_t = \lambda_A\left(1 - \frac{\partial P_L}{\partial P_{td}}\right)$
$P_t = -P_t]_f + \frac{C_t - C_t]_f}{\frac{\partial C_t}{\partial P_t}]_f}$
$P_{Lt} \leq P_t \leq P_{Ht}$
$P_{td} = P_t, P_{Lt}$ OR P_{Ht}
$\sum_A P_{td} = \sum_A P_{td} + P_{td}$
STORE P_{td} AND $\sum_A P_{td}$
Y
84
CONTINUE
N
91
$\lambda_A = \lambda_A + \Delta\lambda_A$
$\Delta\lambda_A = 0.5\Delta\lambda_A$
86
$\left|\sum_A P_{gd} + \sum_A P_{td} - \sum_A P_{ga} - \sum_A P_{ta}\right| < \epsilon$
Y
96
CONTINUE
Y
N
N
$\left(\sum_A P_{gd} + \sum_A P_{td}\right) > \left(\sum_A P_{ga} + \sum_A P_{ta}\right)$
Y
89
CONTINUE
88
Y
N
98
$\sum_{P+A} P_{td} = \sum_A P_{td} + \sum_P P_{ta}$
$\lambda_A = \lambda_A - \Delta\lambda_A$
$\Delta\lambda_A = 0.5\Delta\lambda_A$
90
ERROR EXIT
94
TO FIG. 3

FIG. 3

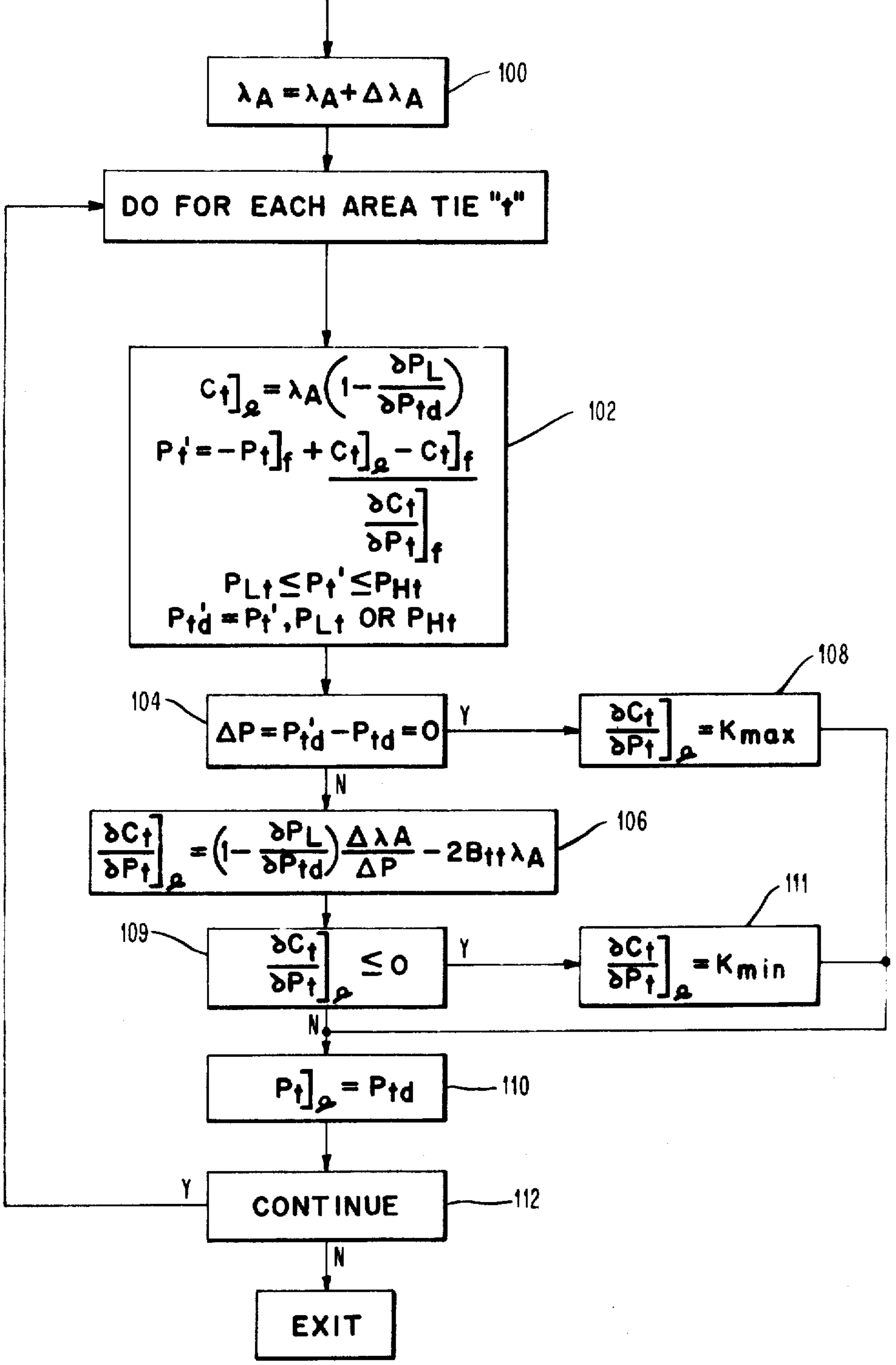

FROM FIG.2 OR FIG. 4
$\lambda_A = \lambda_A + \Delta\lambda_A$
100
DO FOR EACH AREA TIE "t"
$C_t]_\ell = \lambda_A\left(1 - \frac{\partial P_L}{\partial P_{td}}\right)$
$P_t' = -P_t]_f + \frac{C_t]_\ell - C_t]_f}{\frac{\partial C_t}{\partial P_t}\Big]_f}$
$P_{Lt} \leq P_t' \leq P_{Ht}$
$P_{td}' = P_t', P_{Lt}$ OR P_{Ht}
102
104
$\Delta P = P_{td}' - P_{td} = 0$
Y
N
$\frac{\partial C_t}{\partial P_t}\Big]_\ell = K_{max}$
108
$\frac{\partial C_t}{\partial P_t}\Big]_\ell = \left(1 - \frac{\partial P_L}{\partial P_{td}}\right)\frac{\Delta\lambda A}{\Delta P} - 2B_{tt}\lambda_A$
106
109
$\frac{\partial C_t}{\partial P_t}\Big]_\ell \leq 0$
Y
N
$\frac{\partial C_t}{\partial P_t}\Big]_\ell = K_{min}$
111
$P_t]_\ell = P_{td}$
110
CONTINUE
112
Y
N
EXIT

FIG. 4

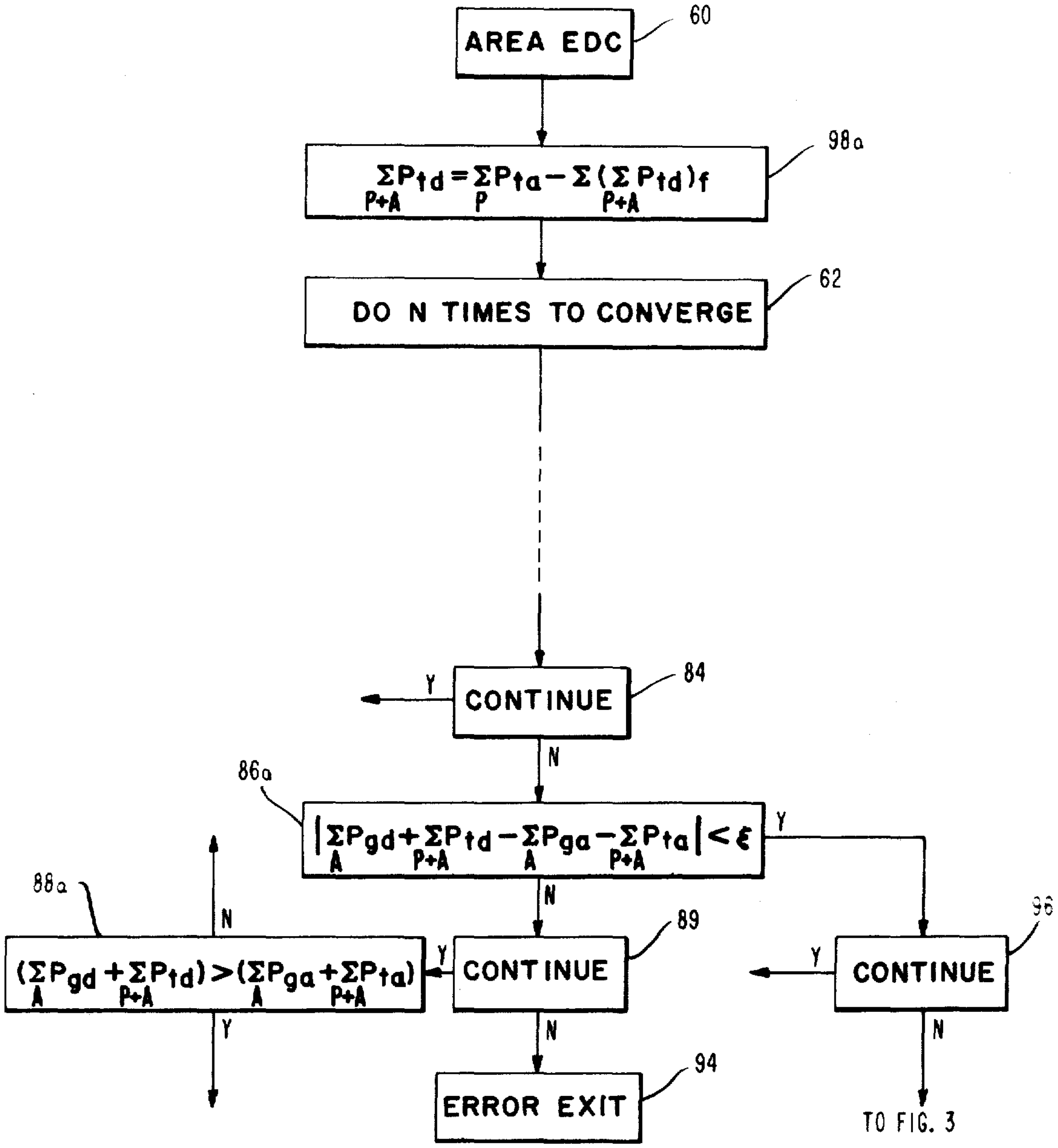
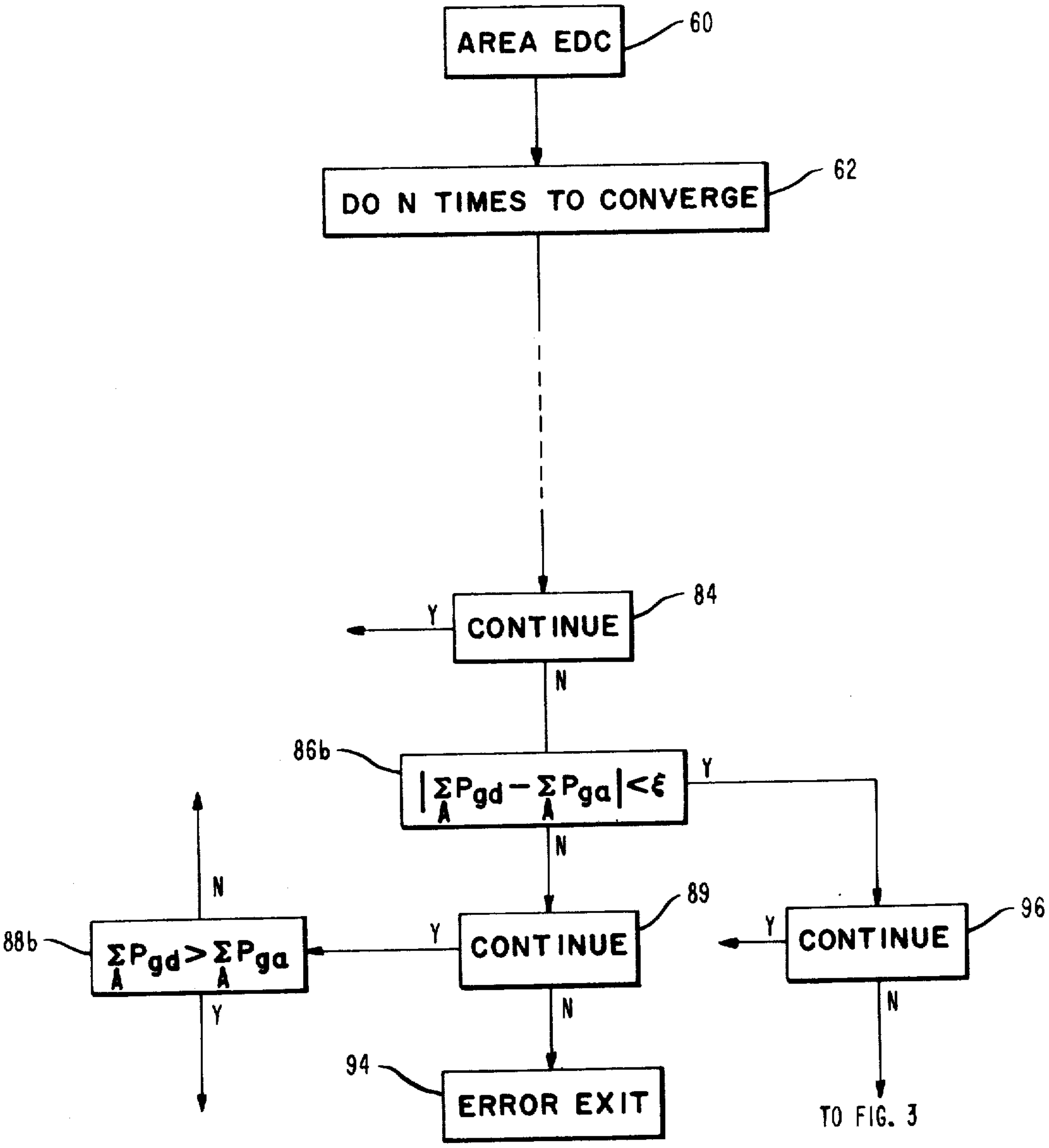

AREA EDC
60
$\sum_{P+A} P_{td} = \sum_{P} P_{ta} - \sum (\sum_{P+A} P_{td})_f$
98a
DO N TIMES TO CONVERGE
62
CONTINUE
84
Y
N
86a
$\left| \sum_{A} P_{gd} + \sum_{P+A} P_{td} - \sum_{A} P_{ga} - \sum_{P+A} P_{ta} \right| < \epsilon$
Y
N
88a
$(\sum_{A} P_{gd} + \sum_{P+A} P_{td}) > (\sum_{A} P_{ga} + \sum_{P+A} P_{ta})$
N
Y
CONTINUE
89
Y
N
ERROR EXIT
94
CONTINUE
96
Y
N
TO FIG. 3

FIG. 5


AREA EDC
60
DO N TIMES TO CONVERGE
62
Y
CONTINUE
84
N
86b
$|\sum_{A} P_{gd} - \sum_{A} P_{ga}| < \epsilon$
Y
N
N
88b
$\sum_{A} P_{gd} > \sum_{A} P_{ga}$
Y
Y
CONTINUE
89
N
Y
CONTINUE
96
N
94
ERROR EXIT
TO FIG. 3

COMPUTING ECONOMIC POWER DISTRIBUTION IN POWER TOOLS

BACKGROUND OF THE INVENTION

This invention relates to a method for the computation of the allocation of generation as between a plurality of generators interconnected in groups to form separate areas with the areas in turn being interconnected by tie lines to form a power pool. More particularly, this invention relates to a method for computing the generation required of each of the generators making up the separate areas to establish for the power pool a minimum total cost for the power generated by the pool for the purpose of obtaining maximum economy of operation while meeting the load requirements of the pool and its scheduled interchange with other pools.

In the past, the computation of the allocation of generation among the separate generators of the interconnected areas of a pool has involved the use of computers which utilize a loss matrix or similar method for introducing into the computations the effect of transmission losses. The parameters of the matrix related to the overall computation problem involved in the pool as contrasted with a loss matrix which utilizes parameters which related only to the particular losses involved in the individual areas of the pool. Thus, in earlier systems such as the system shown in U.S. Pat. No. 3,400,258, issued to one of the present inventors on Sept. 3, 1968, means have been described for the calculation of the desired generation for each of the generating stations of the separate areas of the pools but by virtue of the incorporation of the constants relating to the transmission losses of the pool in a single matrix in addition to the incorporation of the loss constant dealing with a particular area in still another matrix there has been a duplication of computational facilities in order to make possible the separate and individual operation of the areas in the pool with predetermined tie line flows between them as compared with the operation of the pool with an economic distribution of the total generation so that the tie line flows between the areas carried out the economic distribution desired.

In some systems for computing the generation at the various generating stations of each area as well as the power interchange between the areas, it has been necessary to utilize means for dealing with a plurality of interconnecting tie lines between some of the individual areas by calculating an average condition for those tie lines as, for example, in the system described in "Economic Control of Interconnected Systems" by Leon K. Kirchmayer, published by John Wiley & Sons, 1959.

Still other systems for calculating the values of generation and the interchange power between the areas have been disclosed, for example, in the Kirchmayer U.S. Pat. No. 3,117,221, issued Jan. 7, 1964, wherein there has been incorporated a computation not only of the transmission losses but also of the cost of wheeling power through an area.

It is, therefore, an object of this invention to provide a novel method for determining the desired generation for the generators in the separate areas interconnected to form a power pool so as to constantly maintain maximum economy of operation for the pool consistant with restrictions on generator and tie line loading.

A further object of this invention is the provision of a novel method for determining the desired generation for the generators of the stations in the separate interconnected areas of the pool as required for maximum economy of operation of the pool while taking into account the transmission losses on the tie lines interconnecting the areas.

A still further object of this invention is the provision of a novel method for establishing signals representing the desired generation for the generators making up the pool as may be required for satisfying the load of the pool while taking into account the transmission losses on tie lines between the areas with the use of a separate computer for each of the areas.

Still another object of this invention is the provision of a means for computing the economic distribution of total generation in a power pool for maximum economy without the utilization of a pool loss matrix or its equivalent.

SUMMARY OF THE INVENTION

In carrying out the present invention there is provided a method for automatically computing the economic distribution of the generation in a group of areas interconnected for the transmission of power therebetween when at least two of these areas are interconnected by a plurality of transmission lines. This method comprises several steps of which the first is the automatic computation of the incremental cost of power at a boundary point on each tie line between that area and the areas interconnected thereto, based on the incremental generation costs and the incremental transmission losses in that area. The method also includes the step of automatically computing in each of the areas interconnected thereto a second incremental cost of power at the same boundary points mentioned above based on the incremental cost of power generation and the incremental cost of transmission losses in the interconnected areas. In addition, there is included the step of automatically comparing for each area, the first and second incremental costs calculated for each of the boundary points as set forth above and then as a final step there is an automatic computation from the results of this comparison, of the magnitude of the power generation required from each of the power sources and the net interarea tie line interchanges on each of the ties between the respective areas as needed to maintain equality between the sum of the total actual generation and the total actual interchange of the area and the sum of the desired generation and the desired tie line interchanges as computed.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a diagrammatic showing of a power pool showing the tie lines between the individual areas and the communications channels needed between the computers of the areas.

FIGS. 2, 3, 4 and 5 are block diagrams of parts of the algorithm to be followed by a digital computer in making the necessary calculations.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

FIG. 1 shows a power pool 10 which includes 4 interconnected areas, namely areas 1, 2, 3 and 4. Area 1 is shown as having transmission lines **12** and **13** connecting it to area **2** for the interchange of power therebetween. Area **1** is also connected by transmission lines **14** and **15** to areas **4** and **3**, respectively. There is also shown a plurality of tie lines **16** and **17** connecting area **4** and area **3** for interchange of power between those areas while the tie line **18** connects area **3** to area **2**. Areas **1**, **2** and **4** have tie lines **20**, **21**, **22** and **23** which connect to other power pools. For example, tie lines **20** and **21** connect area **1** to an external power pool and the tie lines **22** and **23** connect areas **2** and **4** respectively to other pools.

As shown in FIG. **1**, each of the areas may incorporate a number of generation sources such as the generator **24** whose actual output P_{ga} is supplied to a station or generator bus **26** at which point the incremental cost of generating power C_g may be determined as will be explained subsequently. The power generated in each of the areas is absorbed by the loads of the areas (not shown) and is also transmitted as required over the individual tie lines interconnecting each individual area to other areas of the pool.

Each of the areas of the power pool **10** is shown as having a computer such as the computer **31** of area **1** whose purpose is to compute the desired generation, P_{gd}, for each of the generators in the area as well as the desired net tie line interchange $\Sigma_{P+A} P_{td}$ for that area. This computation in each of the computers **31–34** is preferably made by a digital computer in accordance with a program following the algorithm to be described subsequently. This computation requires that each of the computers have as an input the measured value of the actual generation for each of the generators in the particular area P_{ga} as well as a measured value for the actual tie line flow over the tie lines into the particular area P_{ta}. In addition to these measured values, which are made in the particular area in which the computer is located (the local area), the computer of each area must receive from the computer of each area to which it is interconnected (the foreign areas) the following information based upon values computed during the last economic dispatch computation made by the computers of the foreign areas:

$P_t]_f$ = the tie line power flow over each tie line to the local area from a foreign area.

$C_t]_f$ = the incremental cost at a tie point on each tie line associated with the flow of the power $P_t]_f$.

$\left.\frac{\partial C_t}{\partial P_t}\right]_f$ = the rate of change of the incremental cost of power flow over each tie line with changes in tie line power flow evaluated in the region of $P_t]_f$.

In addition it will be necessary to introduce into the computers **31–34** measured values of the sum of the tie line interchanges between the particular area and the interconnected areas of the pool $\sum_A P_{ta}$ which for area **1** in pool **10** of FIG. **1** is the summation of the tie line flows over the ties **12–15**. Also, it is necessary to sum up the tie line flows to other pools as, for example, by measuring $\sum_P P_{ta}$ which in FIG. **1** is shown as including the measurement of the tie line power flow over the ties **20–23**.

As will be described subsequently, it will be necessary to calculate the incremental cost C_t of power at the tie points on the interarea ties such as shown on tie line **12**. The points at which these costs may be calculated can be at a particular bus to which the tie is connected or at other points, wherever it is convenient for purposes of power system operation.

It will be noted that each of the computers **31–34** has communication channels shown by dashed lines interconnecting the computers in the areas which are interconnected by tie lines. Over the communication channels **41**, **42** and **43** the necessary information is sent from the respective computers **32**, **33** and **34** to computer **31** while over the channels **41** and **42** similar information is sent back from computer **31** to computers **32** and **33**. The communications over channel **43** supplies information between computer **34** and computer **31**. Computer **34** is in the reference area, namely area **4**. Communications channels **46** and **47**, respectively, connect the computers **34** and **33** in one case and **33** and **32** in the other case with communications over both channels going in both directions. Thus, as will be evident from FIG. **1**, it will be necessary to have communication channels between all areas which have interconnecting tie lines between them for the purpose of interchanging the information described above.

Fundamental economic theory has demonstrated that the incremental cost (dollars/MWh) at any point on a power system must be the same when computed from all connecting areas for the system to be in economic balance. This principle has been applied to generators within an area as, for example, in U.S. Pat. No. 2,836,730 issued to E. D. Early on May 27, 1958 and U.S. Pat. No. 2,836,731 issued to W. G. Miller, Jr. on May 27, 1958. Likewise, the principle can be applied to the interchange among areas as illustrated in U.S. Pat. No. 3,117,221 issued to L. K. Kirchmayer on Jan. 7, 1964 and also illustrated in U.S. Pat. No. 3,400,258 issued to W. O. Stadlin, one of the present inventors, on Sept. 3, 1968. Further theoretical background for this proposition may be found in the previously mentioned publication "Economic Control of Interconnected Systems," by Leon K. Kirchmayer.

For control of power interchange between areas of an interconnection to be advantageous to the areas, it is necessary for the incremental costs of power at the boundary of interconnected areas to be the same when it is calculated from either of the interconnected areas. In general an area's incremental cost at the boundary will increase with increased power flow out of the area because of the necessity for increasing its total generation. For the purposes of the computations described herein, the functional relationship between the incremental cost and the tie line power flow can be considered monotonic though it need not necessarily be considered linear. It will be evident that optimum operation of the power pool **10** of FIG. **1** can be defined as that operation which causes an interchange over the tie lines within the pool as well as over the tie lines to external pools as necessary to produce the greatest monetary benefits for the areas of the pool, that is, areas **1–4**.

For the purpose of computing in each of the areas the desired generation for each of the generators in the area and the desired power interchange over the ties to the area, it is advantageous to make the computations in the area by treating each of the tie lines to the area in a manner equivalent to the treatment given to a generator in the area. Thus, if a particular area views an interconnecting tie line it has with another area as being equivalent to a generator in its area whose cost function is determined by the interconnected area, then the desired power flow on the interconnecting tie line can be computed by comparing the incremental cost of power at the tie point with the cost of other sources of energy, that is the generators in its own area as well as with the cost at other tie points. Thus, for each area, optimum operation within an area is achieved when all sources are supplying power at the same values of incremental cost of delivered power, that is, the cost of power at the hypothetical load center of the area.

For the purpose of computing in each of the computers 31–34 of FIG. 1 the desired generation of each of the generators of the separate areas as well as the desired net interchange on the tie lines of the areas, the computers may advantageously be programmed so as to carry out the steps of computation set forth in FIGS. 2, 3 and 4, which will now be explained in detail.

In FIG. 2 the economic dispatch computation program is entered periodically, for example, every 5 minutes as indicated by block 60. The first block following the block 60 in the flow chart of FIG. 2 is block 62 which indicates that the iteration carried on by the program through the outer loop of the flow diagram is to be carried on "N" times in order to converge to a solution. As indicated by block 64, the computation of the incremental transmission losses associated with power from each of the generators of the area as well as the tie lines of the area $\partial P_L/\partial P_n$ is calculated for each of the inputs to the loss matrix. This calculation which is carried out in accordance with the equation shown in block 66 is a well known calculation and is referred to, for example, on page 49 of the above-mentioned Kirchmayer book, "Economic Control of Interconnected Systems" and on page 75 of "Control of Generation and Power Flow on Interconnected Systems," by Nathan Cohn, published by John Wiley & Sons, Inc. in 1966. In the equation in block 66, B_{nm} is the appropriate transmission loss coefficient while B_{n0} is the transmission loss associated with zero power from the source being considered while P_m represents the output of each of the power sources or transmission lines being considered in computing the transmission losses.

As indicated by the block 68, the computation of block 66 is continued for each of the loss matrix inputs and after the transmission loss associated with each of the inputs is calculated, the program then continues to the step indicated in block 70, namely the setting of λ_A to 8 and the setting of $\Delta\lambda_A$ to 4, λ_A being the incremental cost of power delivered to the hypothetical load center of the area in which the computation is being made. The values 8 and 4 for λ_A and $\Delta\lambda_A$ respectively are to be considered as typical and may be altered to extend or contract the range of λ_A in the solution. For example, if we assume that the particular program being discussed is being carried out by computer 31 of FIG. 1, then λ_A the incremental cost of delivered power for area 1.

Having set the value of λ_A and $\Delta\lambda_A$, the next step in the program is to enter a series of computations which are repeated "M" times, as indicated by the block 72. The value of M is chosen in accordance with the desired accuracy of solution and may be typically set to 20. These computations include first a setting equal to zero of the total desired generation of the generating sources ΣP_{gd} as well as a setting to zero of the value of ΣP_{td}, that is the sum of the desired tie line flow over all of the tie lines between the areas interconnected to the area for which the computation is being made. These latter settings are indicated in block 74.

Having made the settings indicated in block 74, the program then enters a portion which is repeated for each of the sources of power, where the sources of power include not only the generators but also the interarea tie lines. That repetition of the computations for each of the sources is indicated by block 76 which is followed by a branching point 78 which causes the program to branch to the series of computations indicated in block 80 if the source of power is a generator or to the series of computations indicated in block 82 if the source of power is an interarea tie line. Normally, the computations relating to the generators as indicated in block 80 will be carried out first. These computations include first a computation of the incremental cost at the station or generator bus for the power provided by the generator. That cost is indicated as C_g and is calculated as a product of λ_A (the incremental cost of delivered power in the area) and the quantity (1 minus the incremental transmission losses) associated with the generation of the particular generator involved.

The generation desired for the generator being considered is a function of the incremental cost of generation at the generator bus, and for a particular incremental cost as calculated by the previous calculation there will be then an associated generation value P_g representing the amount of generation required to provide power at the cost figure C_g. The next step in the computation is to compare the level of generation associated with the computed cost figure C_g with the generator's high and low limits, that is, determine whether P_g is greater than or equal to the low limit P_{Lg} or less than or equal to the high limit P_{Hg}. If P_g is beyond one of the limits, then the desired generation P_{gd} will be set to equal either P_{Lg} or P_{Hg}, depending upon which limit is exceeded; otherwise, the desired generation P_{gd} will be set to equal P_g which was previously computed as the function of the incremental cost C. The value P_{gd} for a particular generator is then added to the total of the values ΣP_{gd} which have been accumulated as a result of the same computations for other generators and the new total, ΣP_{gd}, as a result of this computation will then be available for summing with the desired generation calculated for the next generator to be considered. The last step of the computation as shown in block 80 is to store (save) the value P_{gd} for the particular generator being considered as well as the sum of the desired generations.

Following the computations set forth in block 80, the program then continues as indicated by block 84 until the computations have been done for each source of power, as indicated by block 76. If we assume that all of the generators have been considered and the calculations of the desired generation for each of them has been determined, then the block 78 will cause the computations in block 82 to be made for each of the interarea tie lines.

The computations for the interarea tie lines include first a computation of the incremental cost at the tie point C_t which is calculated in a similar fashion to the calculation for the costs at the generator bus except that the incremental loss quantity is computed with regard to the tie line being considered. After the cost C_t is computed, it is then necessary to determine the power flow P_t over the tie line which will provide the level of power flow associated with the calculated cost C_t. In order to make this calculation, it is necessary to utilize information transmitted from the area at the other end of the particular tie line being considered; thus, as shown by the second equation in block **82**, the value of P_t is determined by adding to the negative of the tie line flow value $P_t]_f$ transmitted from the foreign area to which the tie interconnects a quantity which is computed by dividing the difference between the cost at the tie point C_t and that which was computed in the foreign area $C_t]_f$ by the rate of change of the tie line cost in the foreign area, namely $\partial C_t / \partial P_t]_f$.

The value of P_t is then compared as is done in block **80** with the lower and higher limits set for the tie line, namely P_{Lt} and P_{Ht}, and the desired tie line power flow P_{td} is then set equal to P_t if P_t is within the limits; otherwise it is set equal to the particular limit which is exceeded.

Thus, P_{td}, the desired power transfer over the tie line being considered, is determined and stored and that value is added to the previously accumulated total $\sum_A P_{td}$ for the tie lines previously considered to get a new total $\sum_A P_{td}$ which is also then stored for purposes of the next computation relating to the tie line power flow.

Once all of the generators and all of the interarea tie lines have been considered and the associated desired values for the generation of the generators and the power flow over the tie lines has been calculated, the computations carried out by blocks **80** and **82** are not continued and the comparison shown in block **86** is made. There the absolute value of the sum of all of the calculated values for the desired generation of the separate generators in the area and the sum of the desired values of all of the interarea tie lines to the area are compared with the measured total actual generation of the generators of the area and the measured total of the tie line power flows between the areas, and if the comparison gives a value which is not less than a small number ϵ which is established as a criteria for the accuracy to which the iteration is to be carried, then the program begins a series of steps which are intended to alter the value of λ_A either in an upper or a lower direction depending upon the direction necessary for convergence of the solution. The next step after that shown in block **86** would be the step shown in block **88** which is carried on as long as the program has not iterated more than "M" times, as shown by block **72**, or in other words as long as the block **89** indicates that the program should continue to iterate the value of λ_A.

The consideration indicated by the steps shown in block **88** is whether the sum of the total desired generation of the generators in the area and the total desired power flow over the interarea tie lines is greater than the actual generation in the area and the actual power flow over the interarea tie lines. If the desired values are greater than the actual values, then, as shown in block **90**, λ_A is decreased by a value $\Delta\lambda_A$. The value of $\Delta\lambda_A$, as computed in block **90** for the next iteration, is computed as one half of the present $\Delta\lambda_A$ whose value may initially be **4**, as indicated by block **70**. λ_A may start at a value of **8** as indicated by block **70**. The program then progresses from block **90** to block **72** and another iteration is carried out. If the total desired values for the generation and the interarea tie line flows are not greater than the actual values, then the value for λ_A is increased by a value $\Delta\lambda_A$. $\Delta\lambda_A$ is altered as in block **90**, and the program progresses to block **72** and is continued until the computation converges sufficiently or has been carried out "M" times as indicated by block **72**.

Should the necessary convergence as tested in block **86** fail to materialize within the "M" times that the iteration is carried out, then block **89** will cause the program to transfer to block **94** which will exit from the program and indicate an error. Upon the occurrence of an error, λ_A will have reached either the upper or the lower value of its range. For the initial value shown in block **70** the range of λ_A would be zero to sixteen dollars per megawatt hour. An error with a correspondingly high value of λ_A would be an indication of either too little generating capacity and/or too little tie line capacity.

When the comparison made in block **86** shows that the iteration of λ_A is completed within the desired accuracy, then the program continues "N" times to convergence utilizing for the computation in block **66** the new values computed for the various sources, that is the generators and interarea tie lines as values for P_m. Actual interarea tie line values P_{ta} could also be used for P_m, the choice depending on the convergence characteristics of a particular power system. Once convergence has been reached, the next step will be that shown in block **98**, namely a summation of the total desired interchange over the interarea ties $\sum_A P_{td}$ with the actual measured interchange over the ties from the area under consideration to external pools, namely $\sum_P P_{ta}$ so as to thereby obtain the total net interchange for the area $\Sigma_{P+A} P_{td}$.

Having obtained the total net interchange for the area the next series of steps in the program as shown by FIG. 3 is for the purpose of determining the information to be sent to the interconnected areas relating to the cost of power at the tie points and the rate of change of that cost with changes in tie line power flow. The first step in that determination is the step shown in block **100** where a new value λ_A is found by summing the λ_A previously stored with the value $\Delta\lambda_A$. After that step, the program goes through the series of calculations now to be described for each of the tie lines "t" between the area in which this computer is operating and the other areas within the pool which are interconnected to it.

A series of computations is carried out as shown in block **102**. The first calculation involves the computation of a tie point cost which is to be transmitted from the local area to the interconnected area to which the tie connects $C_t]_l$. That cost is obtained by computing the product of λ_A, as obtained in block **100**, times the quantity (1 minus the transmission losses over the tie line).

Knowing the cost just calculated, it is possible to compute a fictitious value for the power flow over the tie P_t' by adding the value $-P_t]_f$, that is the value received from the interconnected area to the quantity obtained by dividing the difference between the local cost $C_t]_l$ and the cost at the same tie point $C_t]_f$ as sent from the other area by the change in cost with respect to the change in tie line flow as determined and sent from the other area, namely $\partial C_t/\partial P_t]_f$ which represents the slope of the cost curve at the interconnected area.

The value P_t' is then compared with the low limit P_{Lt} and the high limit P_{Ht} and if the value P_t' is not beyond either of the limits, then the value P_{td}' which represents a fictitious desired tie line power flow will be set equal to P_t'; whereas if P_t' exceeds one of the limits, P_{td}' will be set to that limit.

Having a fictitious value P_{td}' determined by changing the value of λ_A by $\Delta\lambda_A$, it is then possible to determine a ΔP by subtracting from the fictitious value P_{td}' the computed desired tie line flow P_{td} and checking to see if that difference is equal to zero, as shown in block **104**. If it is not, the computation shown in block **106** is carried out. In block **106** the computation involves a determination of the change in cost at the tie points with changes in tie power flow as determined from the area at which the computer is located, namely $\partial C_t/\partial P_t]_l$ by multiplying the quantity (1 minus the incremental transmission loss) by the quantity $\Delta\lambda_A$ and dividing by ΔP and then subtracting 2 times the constant B_{tt} (which represents the self constant of the tie as it relates to tie line losses) and also multiplying by λ_A.

If the quantity ΔP is equal to zero and thus the statement in block **104** is true, then the incremental cost change calculated for the local area, as shown in block **108**, is set at a maximum value K_{max} and the program proceeds to block **110** where the value for the tie line power flow calculated at the local area $P_t]_l$ is set equal to P_{td} and as indicated by the block **112**, the computation is then repeated for another tie line.

Once the computation in block **106** is made, the next step after that computation is to determine whether or not the calculated incremental cost at the tie point, as computed in block **106**, was equal or less than zero, as shown in block **109**. If the value was less than or equal to zero, then the incremental cost in the local area would be set to a minimum value of K_{min}, as shown in block **111** and the program would progress into block **110**; whereas if the value of the incremental tie point cost was not equal to or less than zero, the program immediately progresses to block **110** and the value of the incremental tie costs for the local area as computed in block **106** is stored as the incremental cost to be sent to the interconnected area along with the sending of the value $P_t]_l$ and the value $C_t]_l$.

From the above description of FIG. **3** it will be evident that by incrementing the value of λ_A the program has computed a cost figure $C_t]_l$ representing the cost of the power provided by this area to the tie point when the power flow is at a value $P_t]_l$ and a comparable incremental change in tie cost $\partial C_t/\partial P_t]_l$ is also transmitted so that those three items of information can be utilized in the interconnected area as a basis for determining the amount of power flow which is desired over the tie line for economic operation.

The above discussion of FIGS. **2** and **3** relates specifically to the computations which would be carried on in the computers in areas **1**, **2** and **3** of FIG. **1**. Similar computations would be carried on in computer **34** of area **4** with the exception of a few minor changes which will now be discussed. Since area **4** is acting as the reference area, it is necessary to take into account in determining the incremental cost the magnitude of the interchange between the reference area and the power pool and the external power pool to which it may be connected as by the tie line **23**. This change will be evident from the modifications of the algorithm shown in FIG. **2** which should be made as indicated in FIG. **4** for the computer **34** of area **4**. In FIG. **4** the blocks **98***a*, **86***a* and **88***a* indicate that those portions of the algorithm have been changed, the block **98***a* being utilized in place of the block **98** of FIG. **2** while the blocks **86***a* and **88***a*, respectively, replace blocks **86** and **88** of FIG. **2**. It will be noted that the block **98***a* is placed in a different part of the flow chart as compared with the block **98** of FIG. **2** for it is advantageous to make the calculation set forth in block **98***a* prior to the entry of the iterative portions of the program where the value calculated in block **98***a* is utilized as, for example, in block **86***a* and **88***a*.

In block **98***a* the desired net interchange of area **4**, that is the net interchange over the interconnecting tie lines **14**, **16**, **17** and **23**, $\Sigma_{P+A}P_{td}$, is computed as the value $\sum_{P} P_{ta}$ representing the actual total net interchange between pool **10** and the external pools as measured over the interconnections **20**, **21**, **22** and **23** minus the quantity $\Sigma(\Sigma_{P+A}P_{td})_f$ which represents the sum of the desired interchange values computed in the foreign areas, that is, the areas interconnected to area **4** and transmitted to area **4** for this computation over additional communication channels not shown in FIG. **1**.

Between block **98***a* and block **86***a*, the steps of the computation necessary in area **4** are the same as those previously described for the other areas.

In area **4**, the determination as to whether or not the incremental cost iteration has been completed is based upon the absolute value of the desired generation of area **4** plus the desired net interchange of area **4** minus the actual generation of area **4** and also minus the actual net interchange of area **4**. As shown in block **86***a*, that absolute value is compared with ϵ and if it is not less than ϵ, the iteration of λ_A continues as stated in block **89** and the test shown in block **88***a* is made.

In block **88***a* the test comprises the comparison of a quantity which is a sum of the desired generation as computed for area **4** and the desired tie line interchange as computed for area **4** with the respective actual values for those quantities. The program then proceeds in a similar fashion as described with regard to FIG. **2**.

The above discussion of FIG. **4** relates specifically to the computations which would be carried on in the computers in area **4** of FIG. **1**. Different computations could be carried on in the computer **34** of area **4** when it is desirable to eliminate the need for additional communication channels by making the minor changes which will now be discussed. The calculations shown in blocks **86** and **88** would be varied from those shown in FIG. **2** and instead would be as shown in FIG. **5**, wherein the block **86***b* and the block **88***b* respectively replace the blocks **86** and **88** of FIG. **2**. As shown in block **86***b*, the test made involves the comparison of the sum of the desired generations as calculated, namely $\sum_{A} P_{gd}$, with the sum of the measured generations of the area generators $\sum_A P_{ga}$. If the absolute value of the difference is less than ϵ, then the next step in the computation would be carried out by block **96** as indicated in FIG. **2**; whereas otherwise the next step would be carried out by block **89** which continues the computation by carrying out the comparison shown in block **88b** where the value $\sum_A P_{gd}$ is compared to the value $\sum_A P_{ga}$ to see if it is greater than that value. In accordance with the results of that comparison the value λ_A is either increased or decreased as previously explained and as set forth in blocks **90** and **91** of FIG. **2**.

Under some conditions it is advantageous to utilize in the pool a computer, at one area only, which acts as a master computer and which receives information from each of the areas of the pool indicative of the actual generation of each of the generators in those areas as well as the tie line flows. With that information this master computer can compute the desired generation values for each of the generators of the separate areas as well as the desired flow over the individual tie lines and that information can be transmitted to the respective areas as well as being utilized in the master computer. Basically, this arrangement allows for the use of one master computer without the necessity of using a loss matrix for all of the tie lines in the pool. It will be evident to those skilled in the art that either of the systems described in detail, namely for each of the areas with their own computer or with systems described with a master computer or satellite computer at the other areas, could be used depending upon which would be most advantageous for the particular pool involved.

In conjunction with the computations set forth above, each of the areas desirably incorporates a load frequency control system which will effectively modify the generation of the generators in the area in accordance with the computed desired values. The load frequency control system may be operated as a permissive control system utilizing the computed (desired) net interchange of the area as a basis for determining whether the determination should be increased or decreased. For example, the desired net interchange for the area can be compared with the actual net interchange for the area and the difference can be modified by the existing frequency deviation in the area so as to produce an area control error sometimes known as area requirement which can be utilized as an input to a master controller. The master controller can then produce signals such as pulses of duration corresponding to the magnitude of the error and those pulses can be selectively allowed to modify the setting of the governor motor of the generators in the area in accordance with the relative value of the computed value of the desired generation P_{gd} for a particular generator as compared with the actual generation P_{ga} of that generator. Such systems are well known and are illustrated in the above mentioned book by Nathan Cohn entitled, "Control of Generation and Power Flow in Interconnected Systems". Particular reference is directed to that portion of the book following page **103** dealing with "Control Executions". P_{gd}, for example, may be used as the base point setting for a generator. Participation settings, which are usually used in conjunction with base points, can be determined on the basis of the units regulating capability and associated economic factors depending upon the nature of the control system desired.

The combination of the computations described by FIGS. **2**, **3**, **4** and **5** and the simultaneous control of the generators in the several areas is important in that a number of the computations involve the use of actual measured values and when the above mentioned computing procedure is combined with an effective control system operable to carry out the desired economic distribution, the computation will be effective to provide accurate values for the desired generation of each of the generators and the desired net interchange for the areas.

What is claimed is:

1. A method of operating a computing system to compute the economic distribution of the load among the power sources in each of a group of areas interconnected for transmission of power therebetween when at least two of the areas are interconnected by a plurality of transmission lines, comprising the steps of

automatically computing a first incremental cost of power at a boundary point on each tie line between each area and the areas interconnected thereto, said computation being based on the incremental costs and the incremental transmission losses in that area,

automatically computing in each of the areas interconnected thereto, a second incremental cost of power at the same boundary points based on the incremental costs and the incremental transmission losses in the interconnected areas,

automatically comparing for each area the first and second incremental costs calculated for each of the boundary points, and

automatically computing in accordance with the results of said comparisons the magnitude of power generation required from each of the power sources and the net interarea tie line interchange for the respective areas to obtain equality between the sum of the combination of the total actual generation and interarea tie line interchange and the combination of the total desired generation and the desired computed interarea tie line interchange.

2. The method of claim **1** in which the computing system includes a computer for each of the areas to make the automatic computations for the areas.

3. The method of claim **1** in which the computing system carries out all of the automatic computations in a single computer.

4. The method of claim **2** in which the computer in each particular area sends to the computers of each of the areas to which it is interconnected a signal representative of the incremental cost of power at the boundary point of each of the individual tie lines to those interconnected areas for the existing level of incremental cost of delivered power for the area.

5. The method of claim **4** in which the computer in each particular area sends to the computer of each of the areas to which it is interconnected a signal representative of the level of power flow associated with the incremental cost sent to the interconnected areas and signals representative of the rate of change of the incremental cost at each of the tie points associated with said level of tie line power flow.

6. A method of operating a computing system to compute the economic distribution of the load among the power sources in each of a group of areas interconnected for transmission of power therebetween when at least two of the areas are interconnected by a plurality of transmission lines, comprising the steps of

automatically computing a first incremental cost of power at a boundary point on each tie line between each area and the areas interconnected thereto, said computation being based on the incremental costs and the incremental transmission losses in the area involved in computation,

automatically computing in each of the areas interconnected thereto, a second incremental cost of power at the same boundary points based on the incremental costs and the incremental transmission losses in the interconnected areas,

automatically comparing for each area the first and second incremental costs calculated for each of the boundary points,

automatically computing from the results of said comparison the magnitude of power interchange required on each of the tie lines to obtain an economic interchange of power,

automatically computing in each area an incremental cost for generating power from each source of the area,

automatically computing in accordance with said last-named costs the associated desired generation for each of the sources,

comparing the sum of the combined total generation and total desired tie line flow for the area with the combination of the total actual generation and the total actual tie line flow for the area, and

adjusting the incremental cost of delivered power for the area to bring said sum towards zero.

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